

Digital Computer Fundamentals

An Introduction

Jonathan Goode

November 9, 2016

What Is A Digital Computer?

- It's just an adding machine
 - Made out of Gates (Logic, not Robert or Bill)
 - Incredibly, Inconceivably, Wicked Fast
 - Programmable

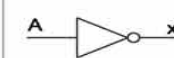






What Are Gates?

- Logic Elements that perform functions
 - And
 - Or
 - Inversion or Negation
 - Often combined into one element, e.g. Nand

The OR Function

- If any input is True (High, or One), the output is True (High, or One)
- If all inputs are False (Low, or Zero), the output is False (Low, or Zero)

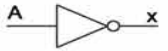






Logic Gates

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The AND Function

- If all inputs are True, the output is True
- If any input is False the output is False

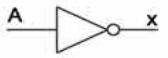






Logic Gates

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The NOT Function (Inverter)

- If the input is True, the output is False
- If the input is False, the output is True

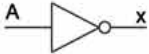






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The NAND Function

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Where's the Adder?

Stay Tuned

- Normal, Decimal Arithmetic vs Binary

- Each column has
 - Two operands, A and B
 - Possible Carry In, Cin
 - Possible Carry Out, Cout

Decimal	
2	5
+3	+6
=5	=11
Binary	
0010	0101
+0011	+0110
=0101	=1011

- Formula for Binary Column

A	0	1	0	1	0	1	0	1
B	0	0	1	1	0	0	1	1
Cin	0	0	0	0	1	1	1	1
Sum	0	1	1	0	1	0	0	1
Cout	0	0	0	1	0	1	1	1

A Little Minimization

Karnaugh Maps

- Main

A	0	1	0	1	0	1	0	1
B	0	0	1	1	0	0	1	1
Cin	0	0	0	0	1	1	1	1
Sum	0	1	1	0	1	0	0	1
Cout	0	0	0	1	0	1	1	1

- Sum

Cin AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

- Cout

Cin AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

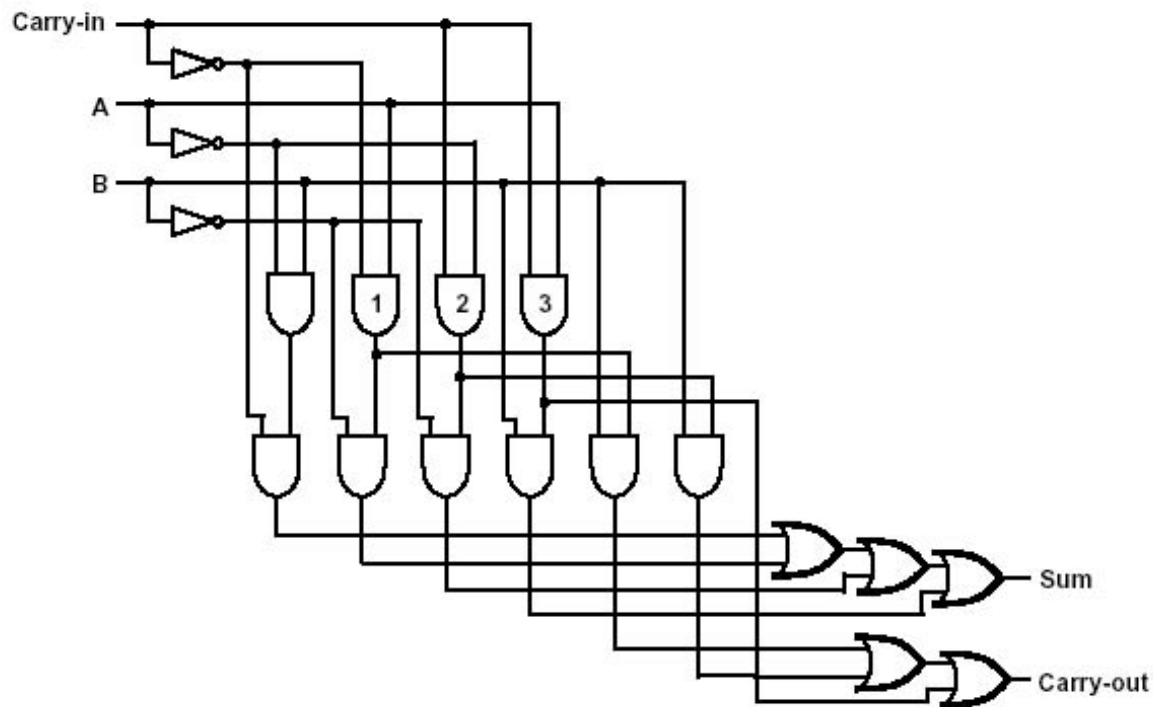
Formula?

		Sum					Cout		
Cin AB	00	01	11	10	Cin AB	00	01	11	10
0	0	1	0	1	0	0	0	1	0
1	1	0	1	0	1	0	1	1	1

- $\text{Sum} = (\text{Not } A \text{ and Not } B \text{ and Cin}) \text{ OR } (\text{Not } A \text{ and } B \text{ and Not Cin}) \text{ OR } (A \text{ and } B \text{ and Cin}) \text{ OR } (A \text{ and not } B \text{ and Cin})$
- $\text{Cout} = A \text{ and } B \text{ OR } ((A \text{ OR } B) \text{ and Cin})$
- Also, $\text{Cout} = A \text{ and } B \text{ OR not } (\text{not } A \text{ and not } B \text{ and Cin})$
- Diagram shows $\text{Cout} = (A \text{ and Cin}) \text{ OR } (A \text{ and } B \text{ and not Cin}) \text{ OR } (\text{not } A \text{ and } B \text{ and Cin})$, also correct and utilizes values already present for Sum

Adder from AND, OR and NOT gates

- Here is a circuit diagram of the adder (from <http://cpuville.com/adder.htm>)



Real Adders

- Real Adders consist of many adders connected in series
 - Sum size, e.g. 32 bits, determines how many stages
 - Inputs A and B are input in parallel, Carries ripple through in series (limiting the speed)

How Fast Is It?

- In 1964, the IBM 7094 had a 2 microsecond cycle time
 - Up to 500,000 operations per second! That's 30 million per minute.
 - This is still hard to fathom, 52 years later
 - Ordinary jet engines run at 10,000 rpm
 - The fastest jet microturbines run at 500,000 rpm, 1/60 of the speed

How Fast Are They Today?

- 3.3 Ghz is typical, 1650 times faster than before
- Multiple cores, pipelining and other techniques increase the speed even more
- Could present details on speedup techniques

Programming

- Computers step through a list of instructions
 - Fetch the instruction from memory, from an address specified in the Instruction Counter
 - Decode and perform specified operation, e.g.
 - CLA CLear and Add (Load) value from memory to Accumulator
 - ADD Add a value from memory to Accumulator
 - STO STOrE the Accumulator into a memory location
 - Test and repeat if not finished

What Could Be Next

- Logic Design
 - Construction using only NAND Gates
 - Negative Numbers
 - Operations: Subtraction, Multiplication, Division
 - Floating Point
- Speed: Modern Computer Architecture
- Programming
 - Machine instruction details
 - Programming Languages
 - Software Development Issues
- What happens when you type on the keyboard – its operation may look like a mechanical typewriter but it's nothing like one